

Fall 2001

Physics 129 Problem Set #3

1. Perkins pg 52 explains rate per target particle = $\Phi \sigma$ so total rate = $d\sigma N_{\text{target}} / \text{Particles}$

We can find N_{target} using the fact that 1 mole of H weighs $1g = 10^{-3} kg$

$$\therefore N_{\text{target}} = \frac{60 \text{ kg}}{\text{m}^3} \times 10^{-4} \text{ m}^3 \times \frac{6.02 \times 10^{23} \text{ molecules}}{10^{-3} \text{ kg}}$$

$$= 3.6 \times 10^{24},$$

$$\text{total rate of } \pi^0 = 10^7 \text{ m}^{-2} \text{ s}^{-1} \times 45 \times 10^{-3} \times 10^{-28} \text{ m}^2 \times$$

$$3.6 \times 10^{24}$$

$$= 162 \text{ s}^{-1}$$

$$\text{rate of } \gamma = 2 \times \text{rate of } \pi^0 = \boxed{324 \text{ s}^{-1}}$$

a) $\frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin \theta} \frac{db}{d\theta} \right|$

$$b = \frac{Q' Q}{2E} \cot \frac{\theta}{2}$$

$$\frac{db}{d\theta} = -\frac{Q' Q}{2E} \frac{1}{\sin^2 \frac{\theta}{2}} \frac{1}{2}$$

$$\therefore \frac{d\sigma}{d\Omega} = \left(-\frac{Q' Q}{2E} \right)^2 \frac{1}{2 \sin^2 \theta/2} \frac{\cos \theta/2}{\sin \theta/2} \frac{1}{\sin \theta}$$

Writing $\sin \theta = 2 \sin \theta/2 \cos \theta/2$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{Q' Q}{4E} \right)^2 \frac{1}{\sin^4 \theta/2}}$$

b) $q^2 = (E - E')^2 - (\vec{p} - \vec{p}')^2$

the problem says we can ignore mass of e
so $E = |\vec{p}|$ $E' = |\vec{p}'|$

$$\therefore q^2 = (E^2 - 2EE' + E'^2) - (p^2 - 2\vec{p} \cdot \vec{p}' + p'^2)$$

$$= -2EE' - 2\vec{p} \cdot \vec{p}' \cos \theta$$

$$= -2EE' (1 - \cos \theta) = 2E^2(\cos \theta - 1)$$

where I have used the fact that target is very heavy to say $E = E'$ for elastic scattering.

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$$dq^2 = 2E^2 d\cos \theta$$

also, $1 - \cos \theta = 2 \sin^2 \frac{\theta}{2}$ so

$$\frac{d\sigma}{dq^2 d\phi} = \frac{1}{2E^2} \left(\frac{Q Q'}{4E} \right)^2 \frac{1}{\left(\frac{q^2}{4E^2} \right)^2}$$
$$= \frac{Q^2 Q'^2}{2q^2}$$

Now integrating both sides

$$\boxed{\frac{d\sigma}{dq^2} = \frac{\pi Q^2 Q'^2}{q^2}}$$

(3)

3a) In this problem we want the relative rate of $A \rightarrow CC$ and $A \rightarrow DD$. We will use Fermi's Golden Rule

$$W = \frac{2\pi}{h} |M|^2 \rho(E)$$

We are told $|M|$ is the same for both processes.

W above is the decay rate at a given E. To get the total decay rate $\Gamma_{A \rightarrow XX}$ ($X = C$ or D) we want to sum over all possible state configurations. Since P and E are continuous variables, this sum becomes an integral. We can write the total sum over final states as an integral over all final state momenta and energies and then use δ functions to impose energy and momentum conservation. So, the sum over final states becomes:

$$\int d^3 p_1 dE_1 d^3 p_2 dE_2 \delta(E_i^2 - p_1^2 - m_i^2) \delta(E_2^2 - p_2^2 - m^2) \delta(E_i - E_1 - E) \delta^3(p_1 - p_f - p_2)$$

here 1 and 2 refer to the final state particles. The first 2 δ functions insure $E^2 = p^2 + m^2$ for each final state particle. The last 2 δ functions impose overall energy and momentum conservation.

(4)

(5)

In the rest frame of A $E_A = m_A$ $\vec{P}_A = (0, 0, 0)$
 so, the sum over final states becomes

$$\int d^3 p_1 dE_1 d^3 p_2 dE_2 \delta(E_1^2 - P_1^2 - m_A^2) \\ \delta(E_2^2 - P_2^2 - m_A^2) \delta(\vec{p}_1 + \vec{p}_2) \delta(m_A - \sqrt{P_1^2 + m_A^2} \\ - \sqrt{P_2^2 + m_A^2})$$

Let's look at each term of the form
 $\int d^3 p dE \delta(E^2 - P^2 - m^2)$:

$$\text{using } dE^2 = 2EdE \Rightarrow dE = dE^2 / 2E$$

this term becomes

$$\int d^3 p \frac{dE^2}{2E} \delta(E^2 - P^2 - m^2) = \int \frac{d^3 p}{2E}$$

$$= \int \frac{d^3 p}{2\sqrt{P^2 + m_A^2}}$$

also, we can do the \int over $\delta(\vec{p}_1 + \vec{p}_2) d^3 p_2$

so our $\int dW$ becomes:

$$\int \frac{d^3 p_1 \delta(m_A - 2\sqrt{P_1^2 + m_A^2})}{4(P_1^2 + m_A^2)}$$

now, use $d^3 p_1 = P_1^2 dP_1 d\Omega$ and $\int d\Omega = 4\pi$

also do change of variables to E

$$E = \sqrt{P^2 + m^2} \Rightarrow dE = \frac{P dP}{\sqrt{P^2 + m^2}} = \frac{P dP}{E}$$

(6)

$$\int dW = \pi \int \sqrt{E^2 - m_A^2} \delta(m_A - 2E) dE \\ = \frac{\pi}{2} \sqrt{\left(\frac{m_A}{2}\right)^2 - m_A^2}$$

$$\therefore \text{relative rate } \frac{\Gamma_{DD}}{\Gamma_{CC}} = \frac{\sqrt{\left(\frac{m_A}{2}\right)^2 - (m_A/8)^2}}{\sqrt{\left(\frac{m_A}{2}\right)^2 - (m_A/4)^2}}$$

$$\frac{\Gamma_{DD}}{\Gamma_{CC}} = \frac{\sqrt{1 - 1/16}}{\sqrt{1 - 1/4}} = \sqrt{\frac{15/16}{3/4}} = \sqrt{5}/2$$

$$\text{b) } \Gamma_{TOT} = \Gamma_{CC} + \Gamma_{DD} = \Gamma_{CC} \left(1 + \frac{\Gamma_{DD}}{\Gamma_{CC}}\right) \\ = \Gamma_{CC} (1 + \sqrt{5}/2) = 2.12 \times 10^{-7} \text{ s}^{-1}$$

$$\tau = \frac{1}{\Gamma_{TOT}} = 4.72 \times 10^{-8} \text{ s}$$

(1)

4. Breit-Wigner with $m = 1.62 \text{ GeV}$ $\Gamma = 150 \text{ MeV}$ and $J=1/2$ eq 2.31 in Perkins gives

$$\sigma = \frac{4\pi \chi^2}{(2s_{\text{tot}})(2s_{\text{rel}})} \frac{M^2/4}{[(E-E_R)^2 + M^2/4]}$$

Peak is at $E = E_R$ $s_a = 0$ $s_b = 1/2$

The only hard part is getting χ , which is the value $\chi = \hbar/p$. p = momentum in center-of-mass frame.

In this frame:

$$P_\mu^\pi = (E_\pi, p_\pi, 0, 0) \quad P_\mu^p = (E_p, -p_\pi, 0, 0)$$

$$\therefore M^2 = E^2 - p^2 = (E_\pi + E_p)^2 = (1.62)^2$$

$$\text{also, } E_\pi = \sqrt{m_\pi^2 + p^2} \quad E_p = \sqrt{m_p^2 + p^2}$$

Solving this exactly doesn't work. So, we will approximate. Note: $M_\pi = 0.140 \text{ GeV}$
 $m_p = 0.938 \text{ GeV}$. Looking at the value of E , we would expect $p < m_p$ and $p \gg m_\pi$. We will solve using this approximation and then check that the solution satisfies the approx.

$$E_\pi \approx p \quad E_p \approx m_p + \frac{p^2}{2m}$$

$$\therefore (p + m_p + \frac{p^2}{2m}) = 1.62$$

$$\frac{p^2}{2m_p} + p + m_p - 1.62 = 0$$

$$p = \frac{-1 \pm \sqrt{(1)^2 + 4(\frac{1}{2m_p})(1.62 - m_p)}}{2(\frac{1}{2m_p})} = 0.53 \text{ GeV}$$

(only keep + solution)

Note: With this soln the approx is valid:

$$E_\pi = \sqrt{(0.53)^2 + (1.1)^2} = 1.11 \text{ N. } 53$$

$$E_p = \sqrt{(0.938)^2 + (0.53)^2} = 1.088$$

$$m_p + p^2/2m = 1.088$$

$$\text{so, } \sigma_{\text{max}} = 4\pi \left(\frac{6.58 \times 10^{-22} \text{ MeVs}}{530 \text{ MeV} / 3 \times 10^8 \text{ m/s}} \right)^2 \frac{(1+1)}{(0+1)(1+1)}$$

$$= \boxed{17.4 \text{ mb}}$$

(2)